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NUMERICAL MODELING OF LAMINAR CIRCULATION FLOW IN A CUBIC CAVITY WITH A MOVING FACE

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Laminar circulation flow in a cubic cavity with a moving face is numerically analyzed. The jet-vortex elements of the flow which develop in the cavity are identified.

1. Genesis of the study of vortex flow in a cubic cavity, induced by the motion of one of its faces with a constant velocity, is associated to a certain extent with the eight-year cycle of works [1–11] on numerical identification of spatial jet-vortex structures generated in the case of flow about an isolated spherical dimple on a plane, in asymmetric uniform flow along a cylinder with a protruding coaxial disk, and in motion of a fluid in a circular vortex cell on one wall of a channel. The study of combined vortex flow in a dimple has been stimulated by the necessity of analyzing the physical mechanism of tornado-like intensification of heat and mass transfer processes in the neighborhood of reliefs consisting of ordered concavities. As a result of the preliminary calculations of a laminar flow in shallow and deep dimples carried out in [1, 2], one has detected curved swirling jet flows that determine the mass transfer within the dimples. In [3–11], systematic numerical investigations of spiral vortex structures generated in a partially confined space of isolated dimples and in the gap between the protruding coaxial disk and the face of a cylinder have been carried out by the method of tagged particles.

This methodological study develops the views of the laws of formation of three-dimensional jet flows built into large-scale vortex structures within the closed space of a cubic cavity. The calculations carried out also supplement results of testing the numerical algorithms of solution of three-dimensional problems; these results have been obtained during the last twenty-five years [12–18].

2. A finite-volume algorithm based on the concept of splitting by physical processes is tested [14, 18]. The system of steady-state, three-dimensional Navier–Stokes equations written in generalized coordinates for increments in dependent variables, for which the Cartesian velocity components are used, is solved using the procedure of pressure correction. In accordance with it, in solving the equations of motion of a medium, at the "predictor" step the preliminary velocity fields are determined for a "frozen" pressure field. Next, at the "corrector" step the field of pressure corrections is first calculated to obey the continuity equation in each cell of the grid, and then the velocity and pressure components are corrected. Thus, all the dependent variables become consistent. Harlow and Welch were among the first to realize the idea of the method of splitting by physical processes in their widely known method of "particles in a cell" in the 1960s. And the concept of pressure correction, which is very efficient for calculations, was described and used by Patankar and Spalding in the early 1970s [14]. As a result, the SIMPLE procedure was developed and it initiated the development of the family of SIMPLE-like algorithms. In this family, noteworthy is the SIMPLEC modification proposed

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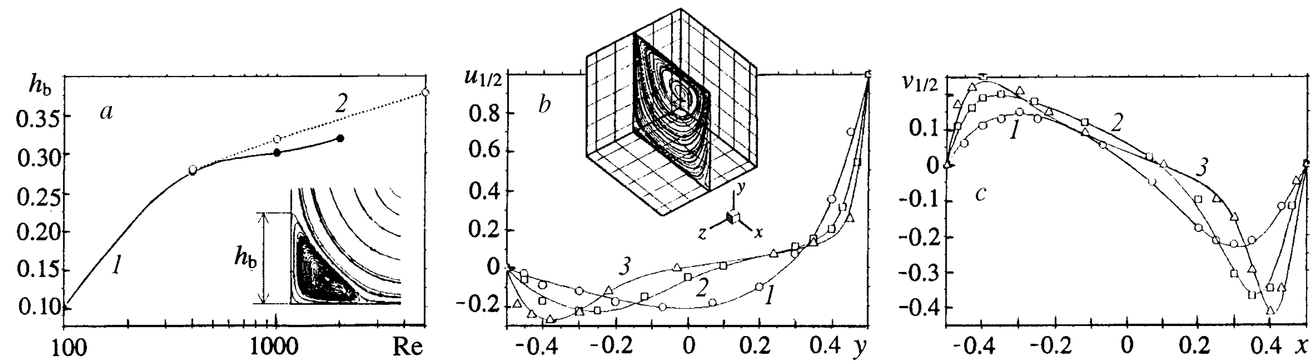

Fig. 1. Comparative analysis of the calculated (1) and experimental (2) dependences of the vertical dimension of the secondary vortex in the middle cross section of a cubic cavity (a) and comparison of the calculated profiles of the horizontal (b) and vertical (c) velocity components in the middle cross section of the cubic cavity at Re = 100 (1), 400 (2), and 1000 (3). The circles denote the data from [15] and the squares and triangles denote the data from [17].

by Raithby and Van Doormaal in the 1980s and meant for a better consistency between the pressure and velocity fields. However, we emphasize that, most probably, because of the savings in computational resources, the methodology developed was based on the components of the velocities rather than on their increments.

The approach described in detail for the first time in [14] significantly extended the acceptability of the SIMPLE algorithms. Its distinguishing characteristic is that it involves construction of schemes of discretization of initial equations relative to the increments in dependent variables. In this case, the finite-volume approximation of the initial differential equation is placed in the explicit part, and in the implicit part a simplified scheme (supplemented with stabilizing terms) of this equation relative to the increments in dependent variables is written. The idea of such a construction is directly related to the algorithms of solution of problems for potential flows. In it, prominence is given to the provision of the most rapid smoothing of the increments. Note that an analogous idea is realized in the multigrid method for accelerating the convergence of the iterative process of solution. It is known that to do this it is necessary to use the most coarse and stable schemes, such as upwind difference schemes with one-sided differences. The quasidiffusion terms with a constant diffusion coefficient, proposed in [14], play a positive part in smoothing high-frequency oscillations in the solutions of equations, characteristic of high Reynolds numbers. It should be noted that the above-described features of this method are unique and they extend the possibilities of control of the computational process.

The discretization of the convective terms of the Navier–Stokes equations is an important aspect because the order of their approximation determines the accuracy of numerical predictions and significantly influences the calculation stability of the computational procedure. It is especially important for modeling circulation flows [14] when the use of the schemes of first order makes the processes of convective transport significantly weaker. In the 1980s, this phenomenon, known as the "effect of artificial or scheme viscosity," was analyzed in detail. As a result, it has been realized that there is no alternative to the use of approximation schemes of high order for representation of convective flows through the faces of a computational cell (for example, the upwind scheme of quadratic interpolation proposed by Leonard). At the present time, its use in commercial products for mathematical modeling of hydrodynamic and thermophysical processes has become standard. We note that this scheme has also been used in the present work.

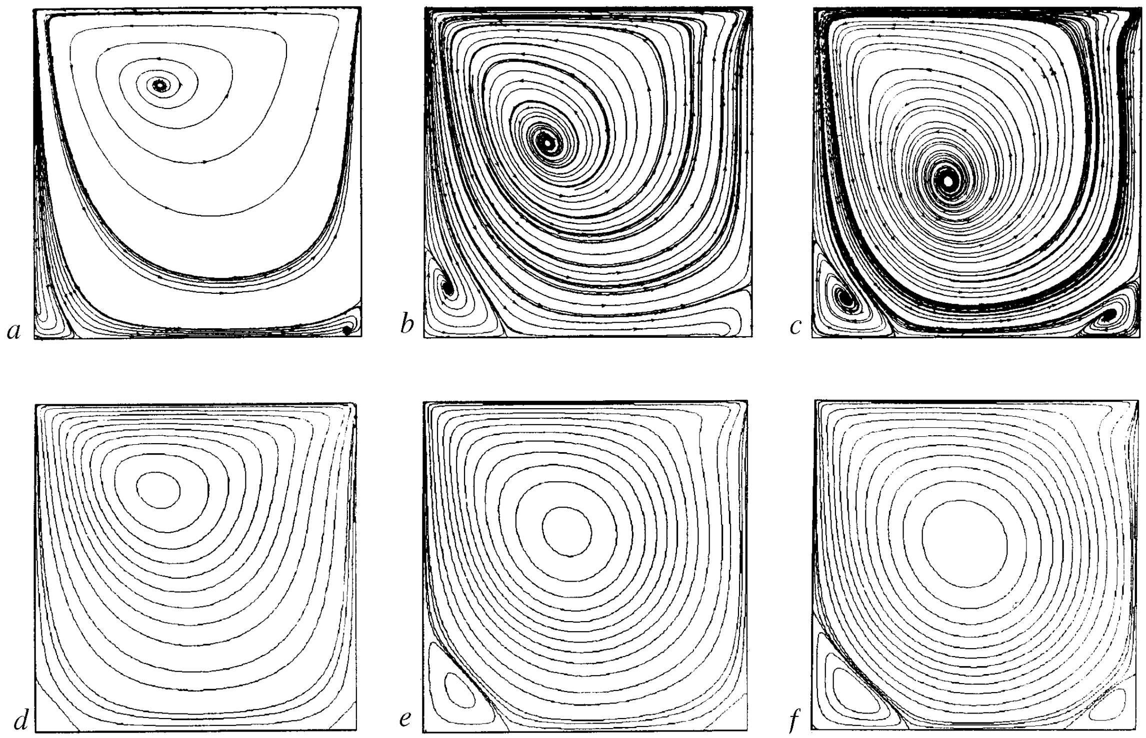

Fig. 2. Patterns of flow in the middle cross section of cubic (a, b, c) and square (d, e, f) cavities at Re = 100 (a, d), 400 (b, e), and 1000 (c, f).

The stability and efficiency of a computational process essentially depend on the method used for solving the difference algebraic equations. There is no doubt that the approach which is used in the present work and is based on incomplete matrix factorization is sufficiently simple and efficient in realization. At the same time, there are assumptions that the method of conjugate gradients [15] is a more powerful computational tool.

3. The calculations of laminar flow of a viscous incompressible fluid in a cubic cavity with a moving face have been carried out for Reynolds numbers from 100 to 2000 on a nonuniform grid containing $40 \times 40 \times 40$ cells. The near-wall step was chosen to be 0.005 in the directions x and y and 0.01 in the transverse direction z. Some of the calculation results are shown in Figs. 1–4.

Comparison of the data obtained in the computational and laboratory experiments [19] has shown that the numerical predictions are sufficiently accurate. The results of the calculations according to the method proposed were also compared to the data of [15, 17] and were found to be in good agreement with them in the velocity profiles in the middle cross section of the cubic cavity for different Reynolds numbers.

A comparative analysis of the evolution of vortex flow in the square cavity and in the middle cross section of the cubic cavity (Fig. 2) has supported the conclusion drawn earlier in [2, 17] that the three-dimensional character of motion of a fluid has a substantial influence on it. An increase in the Reynolds number leads to a marked intensification of the vortex flow in the cavity, which enhances the intensification of the circulation flow (Fig. 1), facilitates the motion of the center of the large-scale vortex to the geometrical center of the cavity, and increases the dimensions of the secondary corner vortices.

Nonetheless, the maximum values of the velocity components of the reverse flow were found to be lower as compared to two-dimensional variants because of the substantial braking influence of side walls (see [14]). Moreover, the structure itself of the primary and secondary vortices in the middle cross section of the cavity is three-dimensional in character, which points to the mass supply in its central zone and mass removal in its corner regions.

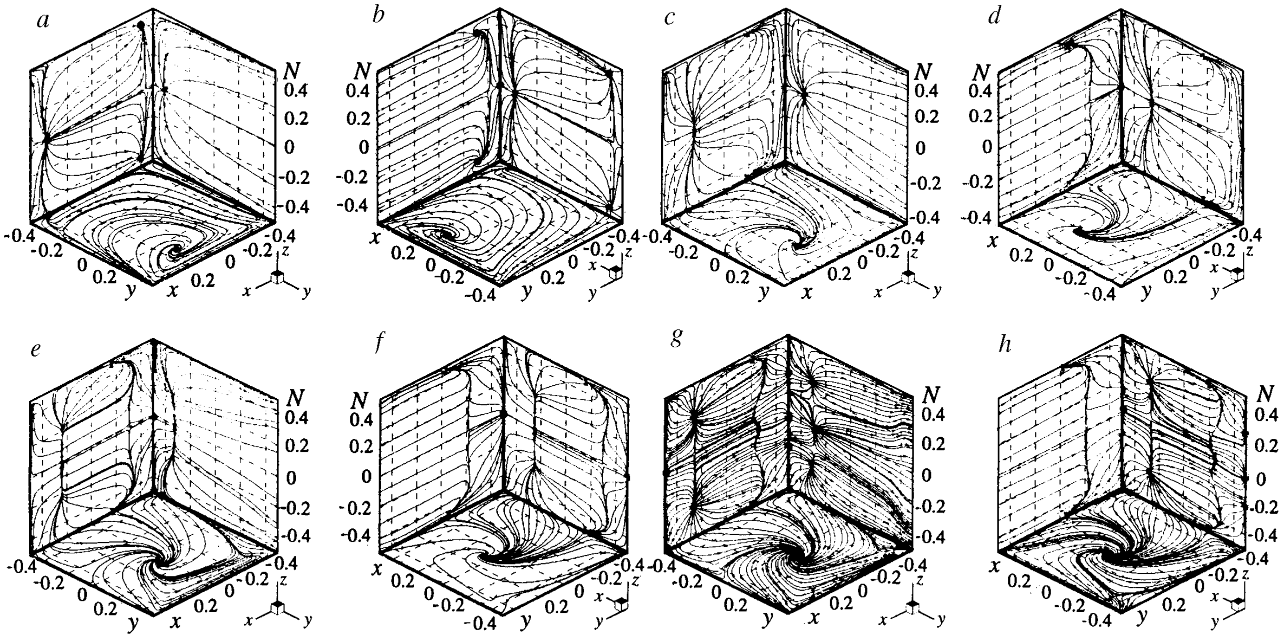

Fig. 3. Patterns of spreading of the fluid over the bottom, side, and front (upstream) faces of a cavity (a, c, e, g) and on the back (downstream), side, and bottom faces (b, d, f, h) at Re = 100 (a, b), 400 (c, d), 1000 (e, f), and 2000 (g, h).

In this investigation as well as in the recent work [17], special emphasis is placed on an analysis of the spreading of the fluid over the walls of a cavity. In this case, the vast possibilities of computer visualization of the flows are realized using the TECPLOT 7.5 package.

The analysis of the patterns of spreading of the fluid over the walls of the cubic cavity, which are given in Fig. 3 and obtained at different Reynolds numbers changing in the range from 100 to 2000, points to the presence of focus-type singular points and lines of flow in them.

Thus, as the Reynolds number increases, the evolution of the pattern of spreading at the bottom of the cavity is accompanied by a decreasing influence of the side walls and transformation of the fluid source at Re = 100 into a line of spreading at Re = 1000. It is quite obvious that the above source is formed when a flow separated in the middle part of the cavity is attached to its bottom face.

One more source arises as a result of the attachment of a near-bottom flow separated from the front wall of the cavity. As the Reynolds number increases, beginning with Re = 1000, the source transforms into a line of spreading.

The interaction of the flows from the above sources, spreading over the bottom of the cavity, occurs along the line of spreading of the fluid. It is significant that this interaction results in the formation of two symmetric peripheral flows at the bottom of the cavity. It is of interest that as the Reynolds number increases, these flows move to the neighborhood of the edges of the side faces.

Beginning with Re = 400, a middle zone of a quasi-two-dimensional separated flow is gradually formed at the bottom of the cavity. At Re = 1000, in this region the streamlines are parallel to each other almost on half of the bottom. Further increase in the Reynolds number to 2000 leads to a very interesting phenomenon characterized by the bending of the streamlines in the central part of the face considered. The reason is that the line of spreading transforms into the line of interaction of two sources at its ends.

Flow of the fluid on the front (upstream) face as a whole is due to the entraining action of the moving wall. However, by analogy with the evolution of spreading of the fluid over the bottom of the cavity

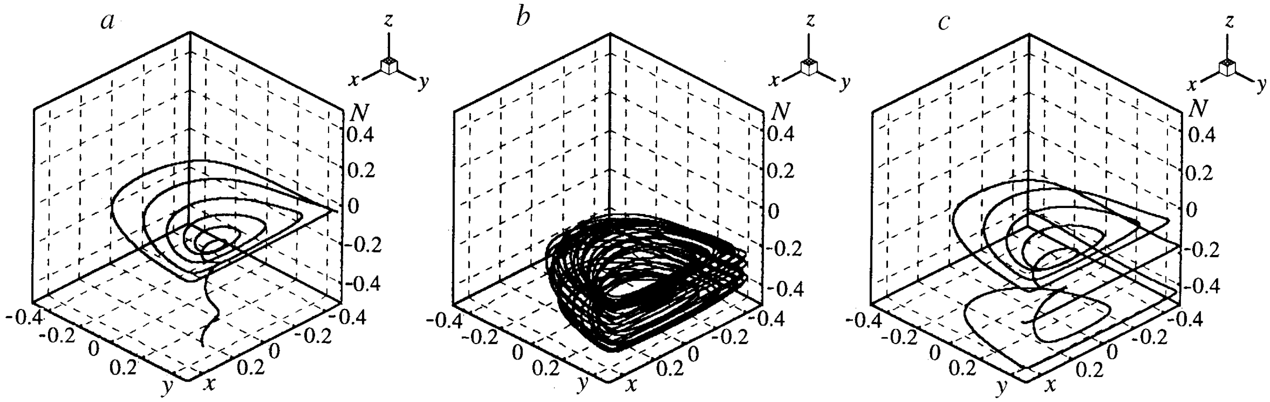

Fig. 4. Tracks of fluid particles in a cubic cavity (Re = 100) introduced into the neighborhood of the focus on the side face (a), in the middle part of the space between the side face and the middle plane (b), and in the corner zone adjacent to the front (upstream) wall (c).

analyzed above, as the Reynolds number increases, the flow from the sources is rearranged to form the line of spreading which indicates the formation of a quasi-two-dimensional vortex in the corner region.

Motion of the fluid on the back (downstream) wall is due to the interaction of the latter and the shear flow formed as a result of motion of the upper cover with a constant velocity. Because of this, the streamlines directed to the bottom of the cavity are practically parallel to the side walls on the larger part of the face considered. However, at the edge near the bottom there arises a source induced by the fluid flowing from the bottom of the cavity to its back wall. The interaction of the flow from this source with the flow incident from above occurs along the line of spreading which becomes parallel to the bottom as Re increases. We can note that two sinks shifting from the bottom as the Reynolds number increases arise in the neighborhood of the side walls. On each side wall of the cavity, a zone of swirling motion with a sink in its central region is formed; this zone moves to the center of the face as the Reynolds number increases.

And finally, we consider the motion of the fluid at high Reynolds numbers. It is known that at Re of the order of 3000 the circulation flow in a cubic cavity becomes unstable, which, first of all, breaks the symmetry of the three-dimensional vortex flow relative to the geometric plane of symmetry. As is seen from the results of the computer visualization of the flow at Re = 2000, the vortex motion in the cubic cavity becomes cellular in character with clearly defined interior interfaces between the flows. This indicates that the flow structure is predisposed to a stability loss.

The introduction of a particle into the neighborhood of the flow on the side wall leads to the formation of a swirling jet flow directed to the middle part of the cavity (Fig. 4a). At low Reynolds numbers (Re = 100) this jet flow turns out to be remarkably built into the large-scale vortex formation shown in Fig. 4b. "The vortex ring or cloud" formed by a single particle of the fluid determines the space zone of the blocked flow, much like the vortex rings are formed in deep dimples on a plane [3, 4]. They can also be called strange attractors.

The tracks of particles introduced into the corner neighborhood of the bottom and of the front wall show an interesting dynamics in the cavity: flowing of the fluid over the side wall to the sink, tornado-like movement of particles to the middle plane, and spinning of particles with their subsequent drift in the peripheral layers to the side wall (Fig. 4c).

By and large, the motion of the fluid in a cubic cavity is similar to the flow in a vortex tube. We should point to the identical features in the behavior of a vortex flow in different types of problems, i.e., external detached flow along a cylinder with a protruding coaxial disk [5, 7], flow about an isolated deep dimple on a plane [1–4, 6, 8, 10, 11], and motion of a fluid in a plane-parallel channel with a circular vortex cell on one wall [9], and in the considered three-dimensional vortex flow in a cubic cavity.

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NOTATION

$\{x, y, z\}$, Cartesian coordinates; h_b, dimension of the secondary vortex; u and v, Cartesian velocity components; Re, Reynolds number. Subscripts: 1/2, parameters in the middle cross section of the cubic cavity; b, middles of the faces of the computational cell.

REFERENCES

1. S. A. Isaev, V. B. Kharchenko, and Ya. P. Chudnovskii, *Inzh.- Fiz. Zh.*, **67**, Nos. 5–6, 373–378 (1994).
2. S. A. Isaev, A. I. Leont'ev, and A. E. Usachov, *Inzh.-Fiz. Zh.*, **71**, No. 3, 484–490 (1998).
3. S. A. Isaev, A. I. Leont'ev, D. P. Frolov, et al., *Pis'ma Zh. Tekh. Fiz.*, **24**, Issue 6, 6–12 (1998).
4. S. A. Isaev, P. A. Baranov, A. E. Usachov, et al., in: *Proc. 8th Int. Symp. on Flow Visualization*, Sorrento, September 1–4 (1998), pp. 217.1–217.8.
5. S. V. Guvernyuk, S. A. Isaev, and A. G. Sudakov, *Zh. Tekh. Fiz.*, No. 11, 138–142 (1998).
6. S. A. Isaev, A. I. Leont'ev, A. E. Usachov, et al., *Izv. Ross. Akad. Nauk, Énergetika*, No. 2, 126–136 (1999).
7. V. K. Bobyshev, S. V. Guvernyuk, and S. A. Isaev, *Inzh.-Fiz. Zh.*, **72**, No. 4, 634–640 (1999).
8. S. A. Isaev, A. I. Leont'ev, and P. A. Baranov, *Pis'ma Zh. Tekh. Fiz.*, **26**, Issue 1, 28–35 (2000).
9. P. A. Baranov, S. V. Guvernyuk, M. A. Zubin, et al., *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 5, 44–56 (2000).
10. S. A. Isaev, A. I. Leont'ev, P. A. Baranov, et al., *Dokl. Ross. Akad. Nauk*, No. 5, 615–617 (2000).
11. S. A. Isaev, A. I. Leont'ev, P. A. Baranov, et al., *Inzh.-Fiz. Zh.*, **74**, No. 2, 62–67 (2001).
12. D. de Vahl and G. D. Mallinson, *Comput. Fluids*, **4**, No. 1, 29–43 (1976).
13. H. S. Ku, R. S. Hirsh, and T. D. Taylor, *J. Comput. Phys.*, **70**, 439–462 (1987).
14. I. A. Belov, S. A. Isaev, and V. A. Korobkov, *Problems and Methods of Calculation of Separating Flows of an Incompressible Fluid* [in Russian], Leningrad (1989).
15. V. I. Pokhilko, *Solution of Navier–Stokes Equations in a Cubic Cavity*, Preprint No. 11 of the Institute of Mathematical Modeling, Russian Academy of Sciences [in Russian], Moscow (1994).
16. B. F. Tyurin and V. B. Kharchenko, *Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh.*, No. 2, 39–44 (1994).
17. T. P. Chiang, W. H. Sheu, and R. R. Hwang, *Int. J. Numer. Meth. Fluids*, **26**, No. 5, 557–579 (1998).
18. S. A. Isaev, P. A. Baranov, N. N. Luchko, et al., *Numerical Modeling of the Separating Flow of an Incompressible Fluid in Square and Cubic Cavities with a Moving Boundary*, Preprint No. 7 of the Heat and Mass Transfer Institute, National Academy of Sciences of Belarus [in Russian], Minsk (1999).
19. J. R. Koseff and R. L. Street, *Trans. ASME, Fluids Eng.* [Russian translation], **106**, No. 4, 299–308 (1984).